An Easie Demonstration of the Analogy of the Logarithmick Tangents to the Meridian Line or sum of the Secants: with various Methods for computing the same to the utmost Exactness, by E. Halley.

IT is now near 100 Years since our Worthy Countryman Mr. *Edward Wright* published his *Correction of Errors in Navigation*, a Book well deserving the perusal of all such as design to use the Sea. Therein he considers the Course of a Ship on the Globe, stearing obliquely to the Meridian; and having shewn, that the *Departure* from the Meridian, is in all cases less than the *Difference* of *Longitude*, in the *ratio* of *Radius* to the *secant* of the *Latitude*, he concludes, That the sum of the *Secants* of each point in the *Quadrant* being added successively, would exhibit a Line divided into Spaces, such as the intervals of the parallels of Latitude ought to be in a true Sea Chart, whereon the Meridians are made parallel Lines, and the *Rhombs* or Oblique Courses represented by right Lines. This is commonly known by the name of the *Meridian Line*, which though it generally be called *Mercators*, was yet undoubtedly Mr. *Wrights* Invention, (as he has made it appear in his Preface.) And the Table thereof is to be met with in most Books treating of Navigation, computed with sufficient exactness for the purpose.

It was first discovered by chance, and as far as I can learn, first publisht by Mr. *Henry Bond*, as an addition to *Norwoods Epitome* of *Navigation*, about 50 Years since, that the *Meridian Line was Analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.* The difficulty to prove the truth of this Proposition, seemed such to Mr. *Mercator*, the Author of *Logarithmotechnia*, that he proposed to wager a good sum of Money, against whoso would fairly undertake it, that he should not demonstrate either, that it was true or false: And about that time Mr. *John Collins*, holding a Correspondence with all the Eminent Mathematicians of the Age, did excite them to this Enquiry.

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The firſt that demonſtrated the ſaid *Analogy*, was the excellent Mr. *James Gregory* in his *Exercitationes Geometricæ*, publiſhed *Anno* 1668. which he did, not without a long train of Conſequences and Complication of Proportions, whereby the evidence of the Demonſtration is in a great meaſure loſt, and the Reader wearied before he attain it. Nor with leſs work and *apparatus* hath the celebrated Dr. *Barrow* in his *Geometrical Lectures* (Lect. XI. App. I.) proved, that the *Sum* of all the *Secants* of any arch is analogous to the *Logarithm* of the *ratio* of *Radius* + *Sine* to *Rad.* — *Sine*, or which is all one, that the *Meridional parts* anſwering to any degree of Latitude, are as the *Logarithms* of the *rationes* of the *Verſed Sines* of the diſtances from both the *Poles*. Since which the incomparable Dr. *Wallis* (on occaſion of a paralogiſm committed by one Mr. *Norris* in this matter) has more fully and clearly handled this Argument, as may be ſeen in Num. 176. of theſe *Tranſactions*. But neither Dr. *Wallis* nor Dr. *Barrow* in their ſaid Treatiſes have any where touched upon the aforeſaid relation of the *Meridian-line* to the *Logarithmick Tangent*; nor hath any one, that I know of, yet diſcovered the Rule for computing independently the interval of the *Meridional parts* anſwering to any two given Latitudes.

Wherefore having attained, as I conceive, a very facile and natural demonſtration of the ſaid Analogy, and having found out the Rule for exhibiting the *difference of Meridional parts*, between any two parallels of Latitude, without finding both the Numbers whereof they are the difference: I hope I may be entituled to a ſhare in the emprovements of this uſeful part of Geometry. Deſiring no other favour of ſome Mathematical Pretenders, than that they think fit to be ſo juſt, as neither to attribute my deſire to pleaſe the Honourable the *Royal Society* in theſe Exerciſes,to any kind of *Vanity* or Love of Applauſe in me, (who too well know how very few theſe things oblige, and how ſmall reward they procure) nor yet to complain *coram non judice*, that I arrogate to my ſelf the Inventions of others, and upon that pretext to depretiate what I do, unleſs at the ſame time, they can produce the Author I wrong, to prove their aſſertions. Such diſingenuity as I have always moſt carefully avoided, ſo I wiſh I had not too much experience of it in the very ſame

perſons

persons, who make it their business to detract from that little share of Reputation I have in these things. But to return to the matter in hand, Let us demonstrate the following *Proposition.*

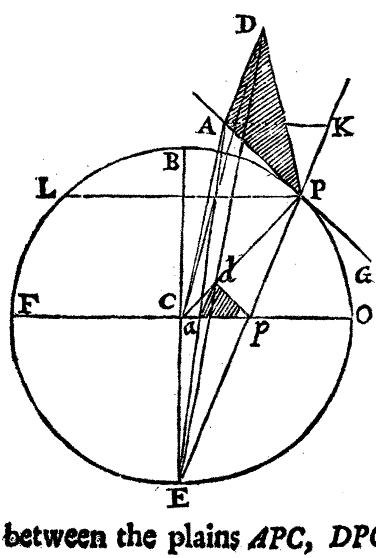
The Meridian Line is a Scale of Logarithmick Tangents of the half Complements of the Latitudes.

For this Demonstration, it is requisite to premise these four *Lemmata.*

Lemma. I. In the *Stereographick Projection* of the Sphere upon the plain of the Equinoctial, the distances from the Center, which in this case is the Pole, are laid down by the Tangents of half those distances, that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3.20.

Lem. II. In the *Stereographick Projection,* the Angles, under which the Circles intersect each other, are in all cases equal to the Spherical Angles they represent: Which is perhaps as valuable a property of this *Projection,* as that of all the Circles of the Sphere thereon appearing Circles: But this not being vulgarly known, must not be assumed without a *Demonstration.*

Let *E B P L* be any great circle of the Sphere, *E* the Eye placed in its Circumference, *C* its Center, *P* any point thereof, and let *F C O* be supposed a plain erected at right Angles to the Circle *E B P L*, on which *F C O* we design the Sphere to be projected. Draw *E P* crossing the Plain *F C O* in *p*, and *p* shall be the point *P* projected. To the point *P* draw the Tangent A P G, and on any point thereof, as A, erect a perpendicular *AD*, at right angles to the plain *EBPL*, and draw the lines *PD*, *AC*, *DC*: and the angle *APD* shall be equal to the Spherical Angle contained between the plains *APC*, *DPC*. Draw also *AE*, *DE*, intersecting

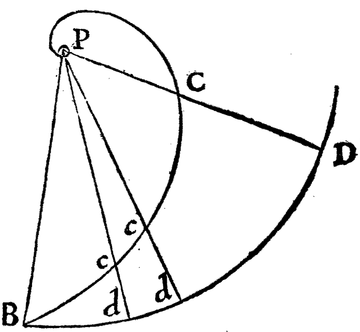


secting the plain *FCO* in the points *a* and *d*; and joyn *a d*, *p d*: I say the Triangle *a d p* is simular to the triangle *ADP*, and the angle *a p d* equal to the angle *APD*. Draw *PL*, *AK* parallel to *FO*, and by reason of the parallels, *a p* will be to *a d* as AK to AD: But (by *Eucl.* 3. 32.) in the triangle AKP, the angle AKP=LPE is also equal to APK=EPG wherefore the sides AK, AP are equal, and 'twill be, as *a p* to *a d* so AP to AD. Whence the angles DAP, *d a p* being right, the angle APD will be equal to the angle *a p d* that is, the Spherical Angle is equal to that on the Projection, and that in all Cases. *Which was to be proved.*

This *Lemma* I lately received from Mr. *Ab. de Moivre*, though I since understand from Dr. *Hook* that he long ago produced the same thing before the *Society*. However the demonstration and the rest of the discourse is my own.

Lemma III. On the *Globe*, the *Rhumb Lines* make equal angles with every Meridian, and by the aforegoing *Lemma*, they must likewise make equal angles with the Meridians in the *Stereographick Projection* on the plain of the *Equator*: They are therefore, in that Projection, *Proportional Spirals* about the Pole Point.

Lemma IV. In the *Proportional Spiral* it is a known property that the angles BPC or the arches BD, are *Exponents* of the *rationes* of *BP* to *PC*: for if the arch BD be divided into innumerable equal parts, right lines drawn from them to the Center P, shall divide the Curve BccC into an infinity of proportionals; and all the lines Pc shall be an infinity of proportionals between PB and PC whose number is equal to all the points d, d, in the arch BD: Whence, and by what I have delivered in Num. 216, it follows, that as BD to Bd, or as the angle BPC to the angle BPc, so is the Logarithm of the *ratio* of PB to PC, to the Logarithm of the *ratio* of PB to Pc.



From these *Lemmata* our Proposition is very clearly demonstrated: For by the first, PB, Pc, PC are the *Tangents* of half the *Complements* of the *Latitudes* in the *Stereographick Projection*: And by the last of them, the differences of Longitude, or angles at the Pole between them, are Logarithms of the *rationes* of those *Tangents* one to the other. But the *Nautical Meridian Line* is no other than a Table of the Longitudes, answering to each minute of Latitude, on the Rhumb-line making an angle of 45 degrees with the Meridian. Wherefore the Meridian Line is no other than a Scale of Logarithmick Tangents of the half Complements of the Latitudes. *Quod erat demonstrandum.*

Coroll. 1. Because that in every point of any *Rhumb Line*, the difference of Latitude is to the *Departure*, as the *Radius* to the *Tangent* of the angle that Rhumb makes with the Meridian; and those equal *Departures* are every where to the differences of Longitude, as the Radius to the Secant of the Latitude; it follows that the differences of Longitude are, on any Rhumb, Logarithms of the same Tangents, but of a differing *Species*; being proportioned to one another as are the Tangents of the angles made with the Meridian.

Coroll. 2. Hence any Scale of Logarithm Tangents, (as those of the Vulgar Tables made after *Brigg*'s form; or those made to *Napiers*, or any other form whatsoever) is a Table of the differences of Longitude, to the several Latitudes, upon some determinate Rhumb or other: And therefore, as the Tangent of the angle of such *Rhumb*, to the Tangent of any other *Rhumb*: So the difference of the Logarithms of any two Tangents, to the difference of Longitude, on the proposed *Rhumb*, intercepted between the two Latitudes, of whose half Complements you took the Logarithm Tangents.

And since we have a very compleat Table of *Logarithm Tangents* of *Brigg*'s form, published by *Vlacq*, *Anno* 1633, in his *Canon Magnus Triangulorum Logarithmicus*, computed to ten Decimal places of the Logarithm, and to every ten Seconds of the Quadrant, (which seems to be more than sufficient for the nicest Calculator) I thought fit to enquire the Oblique angle, with which that Rhumb Line crosses the Meridian, whereon the said Canon of *Vlacq* precisely answers to the differences of Longitude, putting Unity for one minute thereof,

thereof, as in the Common Meridian Line. Now the *momentary augment* or *fluxion* of the Tangent Line at 45 degrees, is exactly double to the *fluxion* of the arch of the Circle, (as may easily be proved) and the Tangent of 45, being equal to *Radius*, the *fluxion* also of the Logarithm Tangent will be double to that of the arch, if the Logarithm be of *Napeirs* form : But for *Brigg's* form it will be as the same doubled arch multiplied into, 0, 43429, &c. or divided by 2, 30258, &c. Yet this must be understood only of the addition of an indivisible arch, for it ceases to be true if the arch have any determinate magnitude.

Hence it appears, that if one minute be supposed Unity, the length of the arch of one minute being ,000290888208665721596154 &c. in parts of the Radius, the proportion will be as Unity to 2,908882 &c. so Radius to the Tangent of 71° 1′ 42″ whose Logarithm is 10. 4637261172071832504 &c. and under that angle is the Meridian intersected by that Rumb Line, on which the *differences* of *Napeirs* Logarithm Tangents of the half Complements of the Latitudes are the true differences of Longitude, estimated in minutes and parts, taking the first Four Figures for Integers. But for *Vlacq*'s Tables we must say.

As .2302585 &c. to 2908882 &c. So Radius to 1,2633114387424456921 2, &c. which is the Tangent of 51° 38′ 9″, and its Logarithm 10,1015104285077209411 62 &c. wherefore in the Rhumb Line, which makes an angle of 51° 38′ 9″ with the Meridian, *Vlacq*'s Logarithm Tangents are the true differences of Longitude. And this compared with our second *Corollary* may suffice for the use of the Tables already computed.

But if a Table of Logarithm Tangents be made by extraction of the root of the Infiniteth power, whose Index is the length of the arch you put for Unity, (as for minutes the ,0002908882th &c. power) which we will call a; such a Scale of Tangents, shall be the true Meridian Line or sum of all the Secants taken infinitely many. Here the Reader is desired to have recourse to my little Treatise of *Logarithms*, published in N° 216. p. 58. that I may not need to repeat it. By what is there delivered, it will follow, that putting t for the excess or defect of any Tangent above or under the *Radius* or *Tangent* of 45; the Logarithm of the

ratio of *Radius* to ſuch Tangent will be

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2}tt + \tfrac{1}{3}ttt - \tfrac{1}{4}tttt + \tfrac{1}{5}t^5, \&c.$$

when the arch is greater than 45gr, or

$$\frac{1}{m} \text{ into } t + \tfrac{1}{2}tt + \tfrac{1}{3}t^3 + \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \&c.$$

when it is leſs than 45gr. And by the ſame doctrine put ting T for the Tangent of any *arch*, and *t* for the difference thereof from the Tangent of another arch, the Logarithm of their *ratio* will be

$$\frac{1}{m} \text{ into } \frac{t}{T} + \frac{tt}{2TT} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$$

when T is the greater Term, or

$$\frac{1}{m} \text{ into } \frac{t}{T} - \frac{tt}{2TT} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$$

when *T* is the leſſer Term:

And if *m* be ſuppoſed ,0002908882, &c. $= a$, its reciprocal $\frac{r}{a}$ will be, 3437,74677078493925 26, &c. which multiplied into the aforeſaid *Series*, ſhall give preciſely the difference of Meridional parts, between the two Latitudes to whoſe half complements the aſſumed Tangents belong. Nor is it material from whether Pole you eſtimate the Complements, whether the elevated or depreſſed; the Tangents being to one another in the ſame *ratio* as their Complements, but inverted.

In the ſame Diſcourſe I alſo ſhewed that the *Series* might be made to converge twice as ſwift, all the even Powers being omitted: and that putting τ for the ſum of the two Tangents the ſame Logarithm would be

$$\frac{2}{m} \text{ or } \frac{2r}{a} \text{ into } \frac{t}{\tau} + \frac{t^3}{3\tau^3} + \frac{t^5}{5\tau^5} + \frac{t^7}{7\tau^7} + \frac{t^9}{9\tau^9}, \&c.$$

but the *ratio* of τ to *t*, or of the ſum of two Tangents to their difference, is the ſame as that of the *ſine* of the ſum of the arches, to the *ſine* of their difference. Wherefore if S be put for the *ſine* Complement of the Middle Latitude, and *s* for the *ſine* of half the difference of Latitudes, the ſame *Series* will be

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \text{ \&c.}$$

wherein as the differences of Latitude are smaller, fewer steps will suffice. And if the Equator be put for the Middle Latitude, and consequently $S = R$, and s to the *sine* of the Latitude, the Meridional parts reckoned from the Equator will be

$$\frac{s}{a} + \frac{s^3}{3rra} + \frac{s^5}{5r^4a} + \frac{s^7}{7r^6a}, \text{ \&c.}$$

which is coincident with Dr. *Wallis*'s solution in Numb. 176. And this same *Series*, being half the Logarithm of the *ratio* of $R + s$ to $R - s$, that is, of the *Versed-sines* of the distances from both Poles, does agree with what Dr. *Barrow* had shewn in his XI. *Lecture*.

The same *ratio* of τ to t may be expressed also by that of *the Sum of the Co-sines of the two Latitudes, to the sine of their difference*: As likewise by that of *the Sine of the Sum of the two Latitudes, to the difference of their Co sines*: Or by that of *the Versed-sine of the Sum of the Co-latitudes, to the difference of the sines of the Latitudes*; Or as *the same difference of the sines of the Latitudes, to the Versed-sine of the difference of the Latitudes*; all which are in the same *ratio* of the Co-sine of the Middle Latitude, to the Sine of half the difference of the Latitudes. As it were easie to demonstrate, if the Reader were not supposed capable to do it himself, upon a bare inspection of a Sheme duly representing these Lines.

This variety of Expression of the same *ratio* I thought not fit to be omitted, because by help of the rationality of the Sine of 30^{gr}, in all cases where the Sum or difference of the Latitudes is 30^{gr}, 60^{gr}, 90^{gr}, 120^{gr} or 150 degrees, some one of them will exhibit a simple *series*, wherein great part of the Labour will be saved: And besides I am willing to give the Reader his choice which of these equipollent methods to make use of; but for his exercise shall leave the prosecution of them, and the *compendia* arising therefrom, to his own industry. Contenting my self to consider only the former, which for all uses seems the most convenient, whether we design to make the whole Meridian Line, or any part thereof, *viz.*

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \text{ \&c.}$$

Wherein a is the length of any Arch which you deſign ſhall be the Integer or Unity in your Meridional Parts; (whether it be a Minute, League or Degree, or any other,) S the Coſine of the Middle Latitude, and s the Sine of half the difference of Latitudes; But the Secants being the Reciprocals of the Co-ſines, $\frac{s}{S}$ will be equal to $\frac{ſs}{rr}$ putting $ſ$ for the Secant of the Middle Latitude; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2ſs}{ar}$ This multiplied by $\frac{ss}{3SS}$ that is by $\frac{ſſss}{3rrrr}$, will give the ſecond ſtep: and that again by $\frac{3ſſss}{5rrrr}$, the third ſtep; and ſo forward till you have compleated as many Places as you deſire. But the ſquares of the *Sines* being in the ſame *ratio* with the *Verſed-ſines* of the double Arches, we may inſtead of $\frac{ss}{3SS}$ aſſume for our Multiplicator $\frac{v}{3V}$, or the Verſed-ſine of the difference of the Latitudes divided by thrice the Verſed-ſine of the ſum of the Co-latitudes, &c. which is the utmoſt *Compendium* I can think of for this purpoſe, and the ſame *ſeries* will become.

$$\frac{2sr}{aS} \text{ into } 1 + \frac{v}{3V} + \frac{v^2}{5V^2} + \frac{v^3}{7V^3} + \frac{v^4}{9V^4}$$

Hereby we are enabled to eſtimate the default of the method of making the Meridian line by the continual addition of the Secants of æquidifferent Archs, which as the differences of thoſe Arches are ſmaller, does ſtill nearer and nearer approach the Truth. If we aſſume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common difference of a rank of Arches: It will be in all caſes, As the Arch of one Minute, to its Chord: : So the Secant of the Middle Latitude, to the firſt ſtep of our *ſeries*. This by reaſon of the near equality between a and $2s$, which are to one another in the *ratio* of Unity to 1—0,0000000352566457713, &c. will not differ from the Secant

Secant $ʃ$ but in the ninth Figure; being less than it in that proportion. The next step being $+\frac{2ʃ^3 s^3}{3ar^5}$ will be equal to the Cube of the Secant of the middle Latitude multiplied into $\frac{2sss}{3arr} = 0{,}000000007051329087 15$; which therefore unless the Secant exceed *ten times Radius*, can never amount to 1 in the fifth place. These two steps suffice to make the Meridian Line or Logarithm Tangent to far more places than any Tables of Natural Secants yet extant, are computed to; but if the third step be required it will be found to be $+ ʃ^5$ into $\frac{2s^5}{5ar^4} = 0{,}0000000000000000089498$; By all which it appears that Mr. *Wright*'s Table does no where exceed the true Meridian Parts by fully half a Minute; which small difference arises by his having added continually the Secants of 1′, 2′, 3′, &c. instead of $0\frac{1}{2}'$, $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, &c. But as it is, it is abundantly sufficient for *Nautical Uses*. That in Sr. *Jonas Moor*'s *New Systeme of the Mathematicks* is much nearer the Truth, but the difference from *Wright* is scarce sensible, till you exceed those Latitudes where Navigation ceases to be practicable, the one exceeding the Truth by about half a Minute, the other being a very small matter deficient therefrom.

For an example easie to be imitated by whoso pleases, I have added the true Meridional Parts to the first and last Minutes of the Quadrant; not so much that there is any occasion for such accuracy, as to shew that I have obtained, and laid down herein, the full *Doctrine* of these *spiral Rhumbs* which are of so great concern in the Art of *Navigation*.

The first Minute is,	1,000000014102658 62178
The Second,	2,000000056410638 06707
The Last, or 89°.59′ is	30374,96343 1141422 8643

and not 32348,5279 as Mr. *Wright* has it, by the addition of the *Secants* of every whole Minute: Nor 30249,8 as Mr. *Oughtred*'s Rule makes it, by adding the *Secants* of every other half Minute. Nor 30364,3 as Sir *Jonas Moor* had concluded it by I know not what method, tho' in the rest of his Table he follow *Oughtred*.

And this may ſuffice to ſhew how to derive the true Meridian Line from the Sines, Tangents or Secants ſuppoſed ready made; but we are not deſtitute of a Method for deducing the ſame independently, from the Arch it ſelf. If the Latitude from the Equator be eſtimated by the length of its Arch A, Radius being Unity, and the Arch put for an *Integer* be a, as before; the Meridional Parts anſwering to that Latitude will be

$$\frac{1}{a}\text{ into } A+\frac{1}{6}A^3+\frac{1}{24}A^5+\frac{1}{\frac{84}{60}}A^7 \text{ or } \frac{61}{5040}A^7 \pm \frac{11}{\frac{2880}{1386}}A^9 \text{ or } \frac{1385}{362880}A^9, \&c.$$

which converges much ſwifter than any of the former *Series*, and beſides has the advantage of A encreaſing in Arithmetical progreſſion, which would be of great eaſe, if any ſhould undertake *de novo* to make the *Logarithm Tangents*, or the Meridian Line to many more places than now we have them. The Logarithm Tangent to the Arch of $45+\frac{1}{2}A$ being no other than the aforeſaid Series $A+\frac{1}{6}A^3+\frac{1}{24}A^5$, &c. in *Napeirs* form, or the ſame multiplied into 0,43429, &c. for *Briggs*'s.

But becauſe all theſe *Series* towards the latter end of the Quadrant do converge exceeding ſlowly, ſo as to render this Method almoſt uſeleſs, or at leaſt very tedious. It will be convenient to apply ſome other Arts, by aſſuming the Secants of ſome intermediate Latitudes; and you may for s or the Sine of α the Arch of half the difference of Latitudes, ſubſtitute $\alpha-\frac{1}{6}\alpha^3+\frac{1}{120}\alpha^5-\frac{1}{5040}\alpha^7+\frac{1}{362880}\alpha^9$, &c. according to Mr. *Newton*'s Rule for giving the Sine from the Arch; And if α be no more than a degree, a very few ſteps will ſuffice for all the accuracy that can be deſired.

And if α be commenſurable to a, that is, if it be a certain number of thoſe Arches which you make your *Integer*, then will $\frac{\alpha}{a}$ be that number: which if we call n. the parts of the Meridional Line will be found to be.

$\frac{sn}{r}$

$$\frac{sn}{r} \text{ into } \left\{ \begin{array}{l} 1 + \frac{ssaa}{3r^4} + \frac{s^4a^4}{5r^8} + \frac{s^6a^6}{7r^{12}}, \&c. \\ - \frac{aa}{6rr} - \frac{ssa^4}{6r^6} - \frac{s^4a^6}{6r^{10}}, \&c. \\ \quad + \frac{1}{120}\frac{a^4}{r^4} + \frac{13s^2a^6}{360r^8}, \&c. \\ \qquad - \frac{1}{5040}\frac{a^6}{r^6}, \&c. \end{array} \right.$$

In this the firſt two ſteps are generally ſufficient for Nautical uſes, eſpecially when neither of the Latitudes exceed 60 degrees, and the difference of Latitudes doth not paſs 30 degrees.

But I am ſenſible I have already ſaid too much for the Learned, tho' too little for the Learner; to ſuch I can recommend no better Treatiſe, than that of Dr. *Wallis* in Numb. 176. wherein he has with his uſual brevity, and that perſpecuity peculiar to himſelf, handled this Subject from the firſt Principles, which here for the moſt part we ſuppoſe known.

I need not ſhew how, by regreſſive work, to find the Latitudes from the Meridional Parts, the Method being ſufficiently obvious. I ſhall only conclude with the propoſal of a Problem which remains to make this Doctrine compleat, and that is this.

A ſhip ſails from a given Latitude, and having run a certain number of Leagues, has altered her Longitude by a given angle. It is required to find the Courſe ſhe ſteared. The ſolution hereof would be very acceptable, if not to the publick, at leaſt to the Author of this Tract, being likely to open ſome further Light into the Myſteries of Geometry.

To Conclude, I ſhall only add, that Unity being Radius, the *Coſine* of the Arch *A*, according to the ſame Rules of Mr. *Newton*, will be

$$1 - \tfrac{1}{2}A^2 + \tfrac{1}{24}A^4 - \tfrac{1}{720}A^6 + \tfrac{1}{40320}A^8 - \tfrac{1}{3628800}A^{10} \&c.$$

from which and the former *Series* exhibiting the *Sine* by the *Arch*, by diviſion it is eaſie to conclude, that the *Natural Tangent* to the Arch *A* is

$$A + \tfrac{1}{3}A^3 + \tfrac{2}{15}A^5 + \tfrac{17}{315}A^7 + \tfrac{62}{2835}A^9 \ \&c.$$

and the *Natural Secant* to the fame Arch

$$1 + \tfrac{1}{2}A^2 + \tfrac{5}{24}A^4 + \tfrac{61}{720}A^6 + \tfrac{277}{8064}A^8 \ \&c.$$

and from the Arithmetick of Infinites, the Number of thefe Secants being the Arch A, it follows, that the fum Total of all the Infinite Secants on that Arch is

$$A + \tfrac{1}{6}A^3 + \tfrac{1}{24}A^5 + \tfrac{61}{5040}A^7 + \tfrac{277}{72576}A^9 \ \&c.$$

the which, by what foregoes, is the *Logarithm Tangent*, of *Napeirs* form, for the Arch of $45^{gr} + \tfrac{1}{2}A$, as before.

And Collecting the Infinite Sum of all the *Natural Tangents* on the faid Arch *A*, there will arife

$$\tfrac{1}{2}AA + \tfrac{1}{12}A^4 + \tfrac{1}{45}A^6 + \tfrac{17}{2520}A^8 + \tfrac{31}{14175}A^{10} \ \&c.$$

twhich will be found to be the Logarithm of the Secant of he fame Arch *A*.

Accounts of Books.

I. *Catoptricæ & Dioptricæ Elementa, Auctore Davide Gregorio,* D. M. *Aftronomiæ Profeffore Saviliano Oxoniæ, & Soc. Reg. Socio,* 8°. *è Theatro Oxon.* 1695.

IN this Treatife the Learned Author demonftrates the Principal Laws of Reflection and Refraction, without reftraining himfelf to any Sect of Philofophers; as alfo the properties of plain and fpherical Surfaces in reflecting and refracting of Rays, and by the way fhews how it comes that fpherical Surfaces produce the fame effects with thofe of certain Spheroids and Conoids, *viz.* becaufe they have the fame degree of Curvature. In the Catoptricks he determines the place of the Image, when the Object and the Eye are not in the fame axis of the reflecting Sphere: an inconvenience that Dioptrical Machines are not fubject to.

Then he proceeds to determine the fituation and bignefs of the Images of fenfibly big Objects, with the

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